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Uwe Hassler und Paulo M. M. Rodrigues

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Uwe Hassler
Darmstadt University of Technology
Macroeconometrics
Residenzschloss
D-64283 Darmstadt
Germany

hassler@vwl.tu-darmstadt.de

Paulo M. M. Rodrigues†
University of Algarve
Faculty of Economics
Campus de Gambelas
8000-117 Faro
Portugal

prodrig@ualg.pt

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Abstract

In this paper, several seasonal unit root tests are analysed in the context of structural breaks at known time and a new break corrected test is suggested. We show that the widely used HEGY test as well as an LM variant thereof are asymptotically robust to seasonal mean shifts of finite magnitude. In finite samples, however, experiments reveal that such tests suffer from severe size distortions and power reductions when breaks are present. Hence, a new break corrected LM test is proposed in order to overcome this problem. Importantly, the correction for seasonal mean shifts bears no consequence on the limiting distributions thereby maintaining the legitimacy of canonical critical values. Moreover, although this test assumes a breakpoint *a priori*, it is robust in terms of misspecification of the time of the break. This asymptotic property is well reproduced in finite samples. Based on a Monte Carlo study, our new test is compared with other procedures suggested in the literature and shown to hold superior finite sample properties.

Keywords: Structural Breaks, Unit Roots, Seasonal Unit Root Tests
JEL: C12, C22

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1 Introduction

The changing nature of the seasonal pattern in economic time series has been a topic of increased interest in recent years. This change (as a result of different frequency unit roots) has led to the development of a considerable number of seasonal unit root tests; see *inter alia* Breitung and Franses (1998), Busetti and Harvey (2000), Canova and Hansen (1995), Dickey, Hasza and Fuller (1984), Hylleberg, Engle, Granger and Yoo [HEGY] (1990), Osborn, Chui, Smith and Birchenhall (1988) and Taylor (2000).

Recently, however, Ghysels (1994) asserted that, "... *testing for seasonal unit roots may face some non-trivial complications, such as the implications of seasonal mean shifts.*" It has been observed that neglected (seasonal) mean shifts can bias unit root tests towards non-rejection (see Perron (1989) and Lopes and Montañés (2000)) or to spurious rejection of the null hypothesis (see Leybourne, Mills and Newbold (1998)). In order to overcome the problems arising from structural breaks in seasonal time series, several new test procedures have been developed. Smith and Otero (1997) and Franses and Vogelsang (1998) adapt the seasonal unit root test procedure proposed by HEGY by employing an approach similar to Perron (1989, 1990) and Perron and Vogelsang (1992), respectively. Other recent test proposals include Franses, Hoek and Paap (1997) who suggest the use of a Bayesian approach and Balcombe (1999) who adapts Zivot and Andrews' (1992) sequential approach to the seasonal context.

Existing literature on breaks in seasonal data can be classified separately according to three distinct approaches: *i*) Smith and Otero (1997) and Franses and Hobijn (1997) allow for a known breakpoint; their analysis parallels Perron (1989, 1990) but lacks asymptotic treatment; *ii*) Franses and Vogelsang (1998) and Balcombe (1999) consider the case of an unknown breakpoint to be estimated from the data, similarly to Zivot and Andrews (1992); *iii*) Here we consider a given breakpoint a priori, that is handled in a manner that does not affect the limiting distribution, in line with the approaches by Park and Sung (1994), Amsler and Lee (1995) and Saikkonen and Lütkepohl (2001). As with Saikkonen and Lütkepohl (1999), our test also displays robustness under misspecification of an assumed breakpoint and may be applied as a test under a break at unknown time. The new test is a variant of the LM-HEGY type test proposed by Breitung and Franses (1998) and Rodrigues (2000), that allows for a linear trend in the data.

In this paper, particular attention is devoted to the procedures indicated in *i*) and *iii*). The paper is organised as follows. Aside from introducing a new test, Section 2 reviews several seasonal unit root test procedures and bridges some existing gaps in the literature. Section 3 provides the limiting distributions of the proposed test with some reference made on the asymptotic implications of ignoring breaks in Section 4. Section 5 compares the new test with competing procedures by means of a Monte Carlo analysis, and Section 6 concludes the paper.

2 Seasonal Unit Root Tests

Consider, under the null hypothesis, the existence of breaks in the seasonal means of size τ_s at time $T_B = \lambda T$, so that

$$y_t = \sum_{s=1}^{4} \delta_s \, D_{s,t} + \frac{\gamma}{4} t + \sum_{s=1}^{4} \tau_s \, D(T_B)_{s,t} + x_t, \text{ with } \Delta_4 x_t = \varepsilon_t, \quad t = 1, 2, \ldots, T, \tag{1}$$

where $D_{s,t}$ are seasonal dummy variables with $D_{s,t} = 1$ when t falls in season s and $D_{s,t} = 0$ otherwise; ε_t is $iid(0, \sigma^2)$; $\Delta_4 = 1 - L^4$, and L is the usual lag operator. Further, $D(T_B)_{s,t} = D_{s,t}$ if $t > T_B$ and $D(T_B)_{s,t} = 0$ otherwise. This representation is typically referred to as the additive outlier (AO) model where the effect of the break occurs abruptly, contrary to the innovative outlier (IO) model where the break has a gradual effect. Equation (1) can still be rewritten as,

$$\Delta_4 y_t = \gamma + \sum_{s=1}^{4} \tau_s \, \Delta_4 D(T_B)_{s,t} + \varepsilon_t,$$

where $\Delta_4 D(T_B)_{s,t}$ is zero for all t but one.

2.1 The HEGY Test

Under the null hypothesis with no breaks, the observed quarterly series y_t is generated as follows,

$$y_t = \sum_{s=1}^{4} \delta_s \, D_{s,t} + \frac{\gamma}{4} t + x_t \text{ with } \Delta_4 x_t = \varepsilon_t, \quad t = 1, 2, \ldots, T. \tag{2}$$

Alternatively, (2) can be written as $\Delta_4 y_t = \gamma + \varepsilon_t$. Based on (2), the HEGY procedure builds on the test regression,

$$\Delta_4 y_t = \sum_{s=1}^{4} \hat{d}_s D_{s,t} + \hat{c}\, t + \hat{\pi}_1 x_{1,t-1} + \hat{\pi}_2 x_{2,t-1} + \hat{\pi}_3 x_{3,t-2} + \hat{\pi}_4 x_{3,t-1} + \hat{\varepsilon}_t \tag{3}$$

for $t = 5, 6, \ldots, T$, where $x_{1,t} = (1 + L + L^2 + L^3) y_t, x_{2,t} = -(1 - L + L^2 - L^3) y_t$ and $x_{3,t} = -(1 - L^2) y_t$.

Under the null hypotheses of unit roots at frequency zero and frequency π, the parameters π_1 and π_2 will be zero, respectively; and under the null of complex unit roots, $\pi_3 = \pi_4 = 0$. Thus, the overall null hypothesis maintained here (that filter $\Delta_4 = (1 - L)(1 + L)(1 + L^2)$ is required to obtain stationarity); assumes that all π_i, $i = 1, 2, 3, 4$, are zero. The nonstandard limiting distributions of the t and F statistics are found in HEGY and Ghysels, Lee and Noh (1994) for the quarterly case; also, Smith and Taylor (1999) provide the asymptotic representations for any seasonal aspect S.

For the quarterly case then, the null hypotheses of interest are,

$$\begin{aligned} H_0^{(1)} \quad &: \quad \pi_1 = 0, \quad H_0^{(2)} : \ \pi_2 = 0, \quad H_0^{(3,4)} : \ \pi_3 = \pi_4 = 0, \\ H_0^{(1,2,3,4)} \quad &\equiv \quad H_0^{(1)} \cap H_0^{(2)} \cap H_0^{(3,4)} \quad \text{and} \quad H_0^{(2,3,4)} \equiv H_0^{(2)} \cap H_0^{(3,4)}. \end{aligned}$$

2.2 The HEGY-AO Test

Motivated by the work of Perron (1989), Smith and Otero (1997) suggest the correction of the HEGY test for seasonal structural breaks at known time. The removal of all deterministics is suggested in a first regression, *viz.*,

$$y_t = \sum_{s=1}^{4} \hat{\delta}_s D_{s,t} + \sum_{s=1}^{4} \hat{\tau}_s D(T_B)_{s,t} + \hat{\gamma}\, t + \widetilde{y}_t \,,$$

where $D(T_B)_{s,t}$ as given in (1). In a second step, the usual HEGY differences are computed from the residuals, $\widetilde{x}_{1,t} = (1+L+L^2+L^3)\widetilde{y}_t$, $\widetilde{x}_{2,t} = -(1-L+L^2-L^3)\widetilde{y}_t$, $\widetilde{x}_{3,t} = -(1-L^2)\widetilde{y}_t$. Hence, the test equation develops the following representation,

$$\Delta_4 \widetilde{y}_t = \hat{\pi}_1 \widetilde{x}_{1,t-1} + \hat{\pi}_2 \widetilde{x}_{2,t-1} + \hat{\pi}_3 \widetilde{x}_{3,t-2} + \hat{\pi}_4 \widetilde{x}_{3,t-1} + \hat{\varepsilon}_t. \tag{4}$$

Smith and Otero (1997, Table 3, p.18) provide critical values based on samples of 1000 observations for different break fractions, λ.

2.3 The HEGY-IO Test

Franses and Hobijn (1997), in contrast to Smith and Otero (1997), consider the innovative outlier model where breaks occur at known time. Using variables from (3), they modify the test regression into,

$$\Delta_4 y_t = \sum_{s=1}^{4} \hat{d}_s D_{s,t} + \sum_{s=1}^{4} \hat{\tau}_s D(T_B)_{s,t} + \hat{c}\, t + \hat{\pi}_1 x_{1,t-1} + \hat{\pi}_2 x_{2,t-1} + \hat{\pi}_3 x_{3,t-2} + \hat{\pi}_4 x_{3,t-1} + \hat{\varepsilon}_t. \tag{5}$$

Franses and Hobijn (1997, Tables 14-18, pp.41-42) tabulate the critical values for several break points based on a sample of size $T = 80$. In practice, we may not know a priori whether the additive or innovative outlier model is correct. Thus, it is important to provide Monte Carlo evidence on the effect of misspecification.

2.4 The LM-HEGY Test

Breitung and Franses (1998) suggest a separate procedure using an LM variant of the HEGY test for general S. However, for the sake of simplicity consideration is given only to the quarterly case, $S = 4$, though results can be generalised to other periodicities as well.

Following Schmidt and Lee (1991, p.285) and Schmidt and Phillips (1992, p.259), the linear trend parameter γ is estimated as the mean of the annual differences, $\overline{\Delta_4 y}$, adopting (2) with no break. Let $\widetilde{x}_t$ denote the demeaned differences so that ,

$$\widetilde{x}_t = \Delta_4 y_t - \overline{\Delta_4 y}$$

and regressors computed recursively as,

$$\widetilde{x}_{1,t} = \widetilde{x}_{1,t-1} + \widetilde{x}_t, \qquad \widetilde{x}_{2,t} = -\widetilde{x}_{2,t-1} - \widetilde{x}_t, \qquad \widetilde{x}_{3,t} = -\widetilde{x}_{3,t-2} - \widetilde{x}_t, \tag{6}$$

where $\widetilde{x}_{k,t} = 0$ for $t \leq 4$ and $k = 1, 2, 3$. Note that this approach follows the approach used to construct the regressors in (3).

We can straightforwardly verify that the HEGY regression corresponding to (21) in Breitung and Franses (1998, p.211), bears the following representation,

$$\widetilde{x}_t = \hat{\pi}_1 \widetilde{x}_{1,t-1} + \hat{\pi}_2 \widetilde{x}_{2,t-1} + \hat{\pi}_3 \widetilde{x}_{3,t-2} + \hat{\pi}_4 \widetilde{x}_{3,t-1} + \hat{\varepsilon}_t, \quad t = 5, 6, \ldots, T \tag{7}$$

whereby the t statistics on $\widehat{\pi}_k$, $k = 1, 2, 3, 4$, is defined as t_{π_k}. In contrast to the HEGY approach, Breitung and Franses (1998, Lemma 3 and 4) suggest the computation of the test statistic, $\Phi_{\pi_3,\pi_4} = t^2_{\pi_3} + t^2_{\pi_4}$, when testing for a complex pair of unit roots. Note that this statistic is asymptotically twice the F statistic for $\pi_3 = \pi_4 = 0$ suggested by HEGY. Breitung and Franses (1998) provide the limiting distributions (Theorem 2, p.209), and critical values (Table 1, p.210), for several sample sizes.

Remark 2.1: Under the null hypothesis of a unit root at frequency π, t_{π_2} has a limiting Dickey-Fuller distribution (the case without intercept and linear trend); see Breitung and Franses (1998, p.208).

Remark 2.2: Though Breitung and Franses (1998) do not propose a test at zero frequency, Rodrigues (2000) shows that the limit of t_{π_1} is equivalent to the limit of the t statistic obtained by Schmidt and Lee (1991, p.287) for a first order autoregressive process. Schmidt and Lee also provide the necessary critical values.

Remark 2.3: Although Breitung and Franses (1998, p.201) indicate that the distributions of their statistics are not affected asymptotically by finite $\tau_s \neq 0$ in (1), finite samples, however, reveal severe size distortions; see Table 1 below.

The results in Schmidt and Lee (1991, Table 2, p.288) and Schmidt and Phillips (1992, p.275) suggest that the power of LM unit root tests may be increased by demeaning the regression variables. Therefore, the following test regression can be adopted in place of (7),

$$\widetilde{x}_t = \sum_{s=1}^{4} \hat{d}_s D_{s,t} + \hat{\pi}_1 \widetilde{x}_{1,t-1} + \hat{\pi}_2 \widetilde{x}_{2,t-1} + \hat{\pi}_3 \widetilde{x}_{3,t-2} + \hat{\pi}_4 \widetilde{x}_{3,t-1} + \hat{\varepsilon}_t. \tag{8}$$

Remark 2.4: As suggested in Rodrigues (2000), defining $t^\mu_{\pi_1}$, $t^\mu_{\pi_2}$ and $F^\mu_{\pi_3,\pi_4}$ as the t and F statistics obtained from (8), $t^\mu_{\pi_1}$, then under the null hypothesis, converges to the limiting distribution obtained by Schmidt and Phillips (1992, p.263) for a first order autoregressive process, while $t^\mu_{\pi_2}$ and $F^\mu_{\pi_3,\pi_4}$ converge to the distributions of the original HEGY test (case with intercept, seasonal dummies and no trend).

2.5 The LM-HEGY-AO Test

By adopting the framework introduced in the previous Subsection, an LM-HEGY test accounting for seasonal breaks can be performed. First, we assume (1) and estimate the linear trend parameter, γ, and the mean shifts, τ_s, from differences by ordinary least squares (OLS),

$$\Delta_4 y_t = \widetilde{\gamma} + \sum_{s=1}^{4} \widetilde{\tau}_s \Delta_4 D(T_B)_{s,t} + \widetilde{x}_t, \quad t = 5, 6, \ldots, T. \tag{9}$$

Second, the residuals are used to compute analogues of the HEGY differences as given in (6), where the starting values are again $\widetilde{x}_{k,t} = 0$ for $t \leq 4$ and $k = 1, 2, 3$. Finally, regression (8) is estimated by OLS according to the new definition of $\widetilde{x}_{k,t}$.

Remark 2.5: The elimination of the four outliers through $\sum \widetilde{\tau}_s \Delta_4 D(T_B)_{s,t}$ in (9) does not affect the limiting distribution because these represent $O_p(1)$ variables; see the analysis in Section 3. In other words, despite corrections for seasonal breaks, t_{π_1}, t_{π_2} and F_{π_3,π_4} statistics from (8) maintain limiting distributions with percentiles tabulated in Schmidt and Phillips (1992, Table 1.A, p.264) and HEGY (1990) (case with intercept, seasonal dummies and no trend) as indicated in Remark 2.4.

Remark 2.6: Moreover, when testing with F statistics, $F_{\pi_1,\pi_2,\pi_3,\pi_4}$ and F_{π_2,π_3,π_4}, for seasonal integration ($H_0^{(1,2,3,4)}$) and for seasonal unit roots ($H_0^{(2,3,4)}$), respectively, the limiting distributions are those characterised in Rodrigues (2000) and Ghysels, Lee and Noh (1994), respectively.

3 Limit Representations of LM-HEGY-AO

3.1 Preliminary Results

Consider the seasonal version of Perron′s crash model given by,

$$y_t = \sum_{s=1}^{4} \delta_s D_{s,t} + \frac{\gamma}{4} t + \sum_{s=1}^{4} \tau_s D(T_B)_{s,t} + x_t, \qquad t = 1, 2, ..., T, \tag{10}$$

$$\text{and } x_t = \alpha x_{t-4} + \varepsilon_t, \; \varepsilon_t \sim iid(0, \sigma^2), \tag{11}$$

where $D_{s,t}$ and $D(T_B)_{s,t}$ are defined below (1). For simplicity, we assume $T_B \bmod 4 = 0$, *i.e.,* the break occurs at the beginning of year $N_B + 1 = T_B/4$. Perhaps a clearer way to analyse this model is to consider its yearly representation using $t = 4(n-1) + s$, where n denotes the year and s the season (see Osborn and Rodrigues (1999) for further details). Hence,

$$y_{s,n} = \delta_s + \gamma(n-1) + \gamma\frac{s}{4} + \tau_s D(N_B)_{s,n} + x_{s,n}, \quad n = 1, 2, \ldots, N = T/4, \tag{12}$$

and

$$x_{s,n} = \alpha x_{s,n-1} + \varepsilon_{s,n} \tag{13}$$

where $D(N_B)_{s,n}$ is an indicator function equal to 1 when $n > N_B$, $s = 1, ..., 4$ and $n = \left[\frac{t-1}{4}\right] + 1$.

Taking the null hypothesis ($\alpha = 1$) in (12) and assuming that y_1 occurs in season $s = 1$ of year 1, we obtain the following representation for $n = 1$

$$y_{s,1} = \delta_s + \gamma \frac{s}{4} + \tau_s D(N_B)_{s,1} + x_{s,0} + \varepsilon_{s,1} \tag{14}$$

and

$$y_{s,n} = \gamma + \tau_s \Delta D(N_B)_{s,n} + y_{s,n-1} + \varepsilon_{s,n}, \quad n = 2, 3, ..., N, \tag{15}$$

where the difference operator now applies to the yearly index, $\Delta x_{s,n} = x_{s,n} - x_{s,n-1}$.

The sum of squared errors entering the log-likelihood function, reached in (10) and (11) when $\alpha = 1$, can be rewritten as,

$$SSE = \sum_{s=1}^{4} \left(y_{s,1} - \delta_s^* - \gamma \frac{s}{4} - \tau_s D(N_B)_{s,1} \right)^2 + \sum_{s=1}^{4} \sum_{n=2}^{N} \left(\Delta y_{s,n} - \gamma - \tau_s \Delta D(N_B)_{s,n} \right)^2;$$

whereby $\delta_s^* = \delta_s + x_{s,0}$, $\Delta y_{s,n} = y_{s,n} - y_{s,n-1}$ denote yearly differences and $\Delta D(N_B)_{s,n} = D(N_B)_{s,n} - D(N_B)_{s,n-1} = 1$ in period s of year $n = N_B + 1$. Here, we assume Gaussianity of the process in order to justify the test as an LM test, though normality is not required for the asymptotic distributions.

Therefore, if we consider $\widetilde{\gamma}$ and $\widetilde{\tau}_s$ as the coefficient estimates obtained from regressing $\Delta y_{s,n}$ on a constant and $\Delta D(N_B)_{s,n}$ then the restricted maximum likelihood estimate of $\widetilde{\delta}_s^*$ is $\widetilde{\delta}_s^* = y_{s,1} - \widetilde{\gamma} - \widetilde{\tau}_s D(N_B)_{s,1}$.

3.2 Asymptotic Results

Only the asymptotic limits of the procedures in the context of breaks of growing magnitude, $\tau_s = \kappa_s N^{\frac{1}{2}}$, are presented, though results for breaks of finite magnitude are obtained by deleting all terms involving κ_s from the asymptotic expressions.

Based on the above results, the following Lemma combines several important convergence results.

Lemma 3.1 *Assuming that the DGP is given by (14) and (15), then as $T = 4N \to \infty$, we can establish from the continuous mapping theorem that,*

$$i) \quad \tfrac{1}{\sqrt{N}} \widetilde{y}_{s,[rN]} \Rightarrow \sigma \left\{ [(W_s(r) - rW(1)] + \left[\tfrac{\kappa_s}{\sigma} d(\lambda, r) - \tfrac{r}{4\sigma} \sum_{s=1}^{4} \kappa_s \right] \right\};$$

$$ii) \quad \tfrac{1}{N^{3/2}} \sum_{n=2}^{N} \widetilde{y}_{s,n} \Rightarrow \sigma \int_0^1 \left\{ V_s(r) + \left[\tfrac{\kappa_s}{\sigma} d(\lambda, r) - \tfrac{r}{4\sigma} \sum_{s=1}^{4} \kappa_s \right] \right\} dr;$$

$$iii) \quad \tfrac{1}{N^2} \sum_{n=2}^{N} \widetilde{y}_{s,n}^2 \Rightarrow \sigma^2 \int_0^1 \left\{ V_s(r) + \left[\tfrac{\kappa_s}{\sigma} d(\lambda, r) - \tfrac{r}{4\sigma} \sum_{s=1}^{4} \kappa_s \right] \right\}^2 dr;$$

$$iv) \quad \tfrac{1}{N} \sum_{n=3}^{N} \Delta \widetilde{y}_{s,n} \widetilde{y}_{s,n-1} \Rightarrow \sigma^2 \int_0^1 \left\{ V_s(r) + \left[\tfrac{\kappa_s}{\sigma} d(\lambda, r) - \tfrac{r}{4\sigma} \sum_{s=1}^{4} \kappa_s \right] \right\} dW_s(r)$$

where '$\Rightarrow$' indicates weak convergence, $\widetilde{y}_{s,n} = y_{s,n} - y_{s,1} - \widetilde{\gamma}\,(n-1) - \widetilde{\tau}_s \left[D(N_B)_{s,n} - D(N_B)_{s,1} \right]$ *(or* $\widetilde{y}_{s,n} = y_{s,n} - \widetilde{\delta}_s^* - \widetilde{\gamma} n - \widetilde{\tau}_s D(N_B)_{s,n}$*),* $\Delta \widetilde{y}_{s,n} = \Delta y_{s,n} - \widetilde{\gamma} - \widetilde{\tau}_s \Delta D(N_B)_{s,n}$*,* $V_s(r) \equiv$

$W_s(r) - rW(1)$, $W(1) = \frac{1}{4}\sum_{s=1}^{4} W_s(1)$, $W_s(r)$, $s = 1, 2, 3, 4$, *are independent standard Brownian motions and* $d(\lambda, r) = 1$ *when* $r > \lambda$.

For proof of this Lemma, see accompanying working paper (Hassler and Rodrigues (2001)).

3.3 The LM-HEGY-AO Test

Following Breitung and Franses (1998), Rodrigues (2000) and the analysis presented in Section 2, the following LM-HEGY type test regression for quarterly data can be proposed,

$$\Delta_4 \widetilde{y}_t = \pi_1 \widetilde{x}_{1,t-1} + \pi_2 \widetilde{x}_{2,t-1} + \pi_3 \widetilde{x}_{3,t-2} + \pi_4 \widetilde{x}_{3,t-1} + \varepsilon_t \tag{16}$$

where $\widetilde{x}_{1,t} = (1 + L + L^2 + L^3)\widetilde{y}_t, \widetilde{x}_{2,t} = -(1 - L + L^2 - L^3)\widetilde{y}_t$ and $\widetilde{x}_{3,t} = -(1 - L^2)\widetilde{y}_t$. Augmentation and seasonal intercepts can also be included if required. Thus, taking into account the results obtained in Section 2, the regressors, $\widetilde{x}_{k,t}$, $k = 1, 2, 3$, can be rewritten as,

$$\begin{aligned}
&\widetilde{x}_{1,t} = \widetilde{y}_{1,n} + \widetilde{y}_{2,n} + \widetilde{y}_{3,n} + \widetilde{y}_{4,n} + O_p(1) \\
&\widetilde{x}_{2,t} = \begin{cases} \widetilde{y}_{1,n} - \widetilde{y}_{2,n} + \widetilde{y}_{3,n} - \widetilde{y}_{4,n} + O_p(1) & t \bmod 2 = 1 \\ -\widetilde{y}_{1,n} + \widetilde{y}_{2,n} - \widetilde{y}_{3,n} + \widetilde{y}_{4,n} + O_p(1) & t \bmod 2 = 0 \end{cases} \\
&\widetilde{x}_{3,t} = \begin{cases} -\widetilde{y}_{1,n} + \widetilde{y}_{3,n} + O_p(1) & t \bmod 4 = 1 \\ -\widetilde{y}_{2-n} + \widetilde{y}_{4,n} + O_p(1) & t \bmod 4 = 2 \\ \widetilde{y}_{1,n} - \widetilde{y}_{3,n} + O_p(1) & t \bmod 4 = 3 \\ \widetilde{y}_{2,n} - \widetilde{y}_{4,n} + O_p(1) & t \bmod 4 = 0 \end{cases}
\end{aligned} \tag{17}$$

where $n = \left[\frac{t-1}{4}\right] + 1$.

Hence, the following Theorem regarding the asymptotic distributions of the t-statistics on the least-squares estimates obtained from (16) can be stated:

Theorem 3.1 *Assuming that the DGP is (10) and (11) with* $\alpha = 1$, *then as* $T \to \infty$, *the continuos mapping theorem carries the following convergence results for the t-statistics on* *the least-squares estimates obtained from the estimation of (16),*

$$t_{\widehat{\pi}_1} \Rightarrow \frac{\int_0^1 \sum_{s=1}^{4} \left\{ V_s^*(r) + \frac{1}{2}\left[\frac{\kappa_s}{\sigma} d(\lambda, r) - \frac{r}{4\sigma} \sum_{s=1}^{4} \kappa_s \right] \right\} d\left(\sum_{s=1}^{4} W_s^*(r) \right)}{\left\{ \int_0^1 \left\{ \sum_{s=1}^{4} \left\{ V_s^*(r) + \frac{1}{2}\left[\frac{\kappa_s}{\sigma} d(\lambda, r) - \frac{r}{4\sigma} \sum_{s=1}^{4} \kappa_s \right] \right\} \right\}^2 dr \right\}^{1/2}} \equiv \mathcal{J}_1, \tag{18}$$

$$t_{\widehat{\pi}_2} \Rightarrow \frac{\int_0^1 \sum_{s=1}^{4} (-1)^{s+1} \left\{ W_s^*(r) + \frac{\kappa_s}{2\sigma} d(\lambda, r) \right\} d\left(\sum_{s=1}^{4} (-1)^{s+1} W_s^*(r) \right)}{\left\{ \int_0^1 \left\{ \sum_{s=1}^{4} (-1)^{s+1} \left\{ W_s^*(r) + \frac{\kappa_s}{2\sigma} d(\lambda, r) \right\} \right\}^2 dr \right\}^{1/2}} \equiv \mathcal{J}_2, \tag{19}$$

$$t_{\widehat{\pi}_3} \Rightarrow \frac{\left\{ \int_0^1 \left\{ W_{13}^*(r) + \frac{\kappa_{13}}{\sqrt{2}\sigma} d(\lambda, r) \right\} dW_{13}^*(r) + \int_0^1 \left\{ W_{24}^*(r) + \frac{\kappa_{24}}{\sqrt{2}\sigma} d(\lambda, r) \right\} dW_{24}^*(r) \right\}}{\left\{ \int_0^1 \left\{ W_{13}^*(r) + W_{24}^*(r) + \frac{\kappa_{13}}{\sqrt{2}\sigma} d(\lambda, r) + \frac{\kappa_{24}}{\sqrt{2}\sigma} d(\lambda, r) \right\}^2 dr \right\}^{1/2}} \equiv \mathcal{J}_3, \tag{20}$$

$$t_{\widehat{\pi}_4} \Rightarrow \frac{\left\{ \int_0^1 \left\{ W_{24}^*(r) + \frac{\kappa_{24}}{\sqrt{2}\sigma} d(\lambda, r) \right\} dW_{13}^*(r) - \int_0^1 \left\{ W_{13}^*(r) + \frac{\kappa_{13}}{\sqrt{2}\sigma} d(\lambda, r) \right\} dW_{24}^*(r) \right\}}{\left\{ \int_0^1 \left\{ W_{13}^*(r) + W_{24}^*(r) + \frac{\kappa_{13}}{\sqrt{2}\sigma} d(\lambda, r) + \frac{\kappa_{24}}{\sqrt{2}\sigma} d(\lambda, r) \right\}^2 dr \right\}^{1/2}} \equiv \mathcal{J}_4 \tag{21}$$

where $W_{ij}(r) = W_i(r) - W_j(r)$, $\frac{\kappa_{ij}}{\sigma} d(\lambda, r) = \frac{\kappa_i}{\sigma} d(\lambda, r) - \frac{\kappa_j}{\sigma} d(\lambda, r)$, $i = 1, 2$, $j = i + 2$; $V_s^*(r) = \frac{\mathbf{V}_s(\mathbf{r})}{2}$, $W_s^*(r) = \frac{\mathbf{W}_s(\mathbf{r})}{2}$, $W_{ij}^*(r) = \frac{\mathbf{W}_{ij}(\mathbf{r})}{\sqrt{2}}$, $i = 1, 2, j = i + 2$, *and* $W_s(r)$ *and* $W_{ij}(r)$ *are independent standard Brownian motions,* $V_s(r) \equiv W_s(r) - rW(1)$ *and* $d(\lambda, r) = 1$ *when* $r > \lambda$.

For Proof see relevant working paper (Hassler and Rodrigues (2001)).

Remark 3.1: Notice that (18) is equivalent to the distribution obtained by Schmidt and Lee (1991, p.263), whereas, (19) to (21) correspond to distributions obtained from a HEGY test regression without deterministics, thus supporting Remark 2.1.

Remark 3.2: If seasonal dummies are included in test regression (16), these distributions change. In this context, Remarks 2.4 and 2.5 apply.

Corollary 3.1 *The joint tests from (16) for seasonal integration* $\left(H_0^{(1,2,3,4)}\right)$, *seasonal unit roots* $\left(H_0^{(2,3,4)}\right)$ *and complex unit roots* $\left(H_0^{(3,4)}\right)$ *have limiting distributions, that are averages of the squared distributions of Theorem 3.1, i.e.,*

$$F_{\pi_1,\ldots,\pi_4} - \frac{1}{4} \sum_{i=1}^{4} \mathcal{J}_i^2 = o_p(1)\,, \quad F_{\pi_2,\ldots,\pi_4} - \frac{1}{3} \sum_{i=2}^{4} \mathcal{J}_i^2 = o_p(1)\,, \quad F_{\pi_3,\pi_4} - \frac{1}{2} \sum_{i=3}^{4} \mathcal{J}_i^2 = o_p(1)\,.$$

For proof of this Corollary, the techniques used by for example, Ghysels, Lee and Noh (1994), Osborn and Rodrigues (1999) and Smith and Taylor (1999) can be adopted.

4 Implications of Ignoring Mean Shifts

In this Section, we analyse the implications that neglected breaks can have on the test procedures, particularly HEGY and LM-HEGY tests. Moreover, we examine the asymptotic effect of a misspecification of an assumed break point on the break corrected LM-HEGY-AO test. Again, only the limits in the context of breaks of growing magnitude, $\tau_s = \kappa_s N^{\frac{1}{2}}$, are presented, however, the results for breaks of finite magnitude derive from deleting all terms involving κ_s from the asymptotic formulae.

4.1 The HEGY Test

Consider that the DGP is given by (14) and (15) with $\delta_s = 0$, $s = 1, ..., 4$ and $\gamma = 0$, for simplicity of notation. Also, equally assume as Leybourne, Mills and Newbold (1998) have, that in order to accept a break of growing magnitude the break must grow with the sample size, that is, $\tau_s = \kappa_s N^{\frac{1}{2}}$. Hence, $y_{s,n}$ can be written as,

$$y_{s,n} = \kappa_s N^{\frac{1}{2}} \sum_{i=2}^{n} \Delta D(N_B)_{s,i} + y_{s,0} + \sum_{i=2}^{n} \varepsilon_{s,i}, \qquad n = 2, 3, ..., N. \tag{22}$$

Consequently, the asymptotic implications of this break are summarised in the following theorem.

Theorem 4.1 *Assuming that the DGP is given by (14) and (15) with $\tau_s = \kappa_s N^{\frac{1}{2}}$, then as $N \to \infty$,*

i) $\quad \frac{1}{N^{\frac{3}{2}}} \sum_{n=2}^{N} y_{s,n} \Rightarrow \sigma \left[\kappa_s \lambda + \int_0^1 W_s(r) dr \right];$

ii) $\quad \frac{1}{N^2} \sum_{n=2}^{N} y_{s,n}^2 \Rightarrow \sigma^2 \left[\kappa_s^2 \lambda + \int_0^1 W_s^2(r) dr + 2\kappa_s \int_\lambda^1 W_s(r) dr \right];$

iii) $\quad \frac{1}{N} \sum_{n=2}^{N} \Delta y_{s,n} y_{s,n-1} \Rightarrow \sigma^2 \left[\kappa_s^2 + \kappa_s W_s(r) + \kappa_s \left[W_s(1) - W_s(\lambda) \right] + \int_0^1 W_s(r) dW_s(r) \right]$

where '$\Rightarrow$' indicates weak convergence, $W_s(r)$, $s = 1, ..., 4$, are independent standard Brownian motions, and λ represents the position where the break occurs in the series.

Remark 4.1: Theorem 4.1 reasserts that the limiting distributions of the HEGY test are affected by breaks of growing magnitude. Deleting the terms involving κ_s, shows that breaks of finite magnitude do not impinge on the distributions. A corresponding result has been proven for the Dickey-Fuller test by Amsler and Lee (1995).

4.2 The LM-HEGY Test

In the case of LM-HEGY tests and given the model in (10) and (11) carrying $\tau_s = \kappa_s N^{\frac{1}{2}}$, we observe that with $\alpha = 1$,

$$\widetilde{\gamma} = \gamma + \frac{N^{1/2}}{4(N-1)} \kappa_s + \overline{\varepsilon}_s$$

and

$$\widetilde{y}_{s,n} = \sum_{k=2}^{n} (\varepsilon_{s,k} - \overline{\varepsilon}_s) + \sum_{k=2}^{n} \kappa_s N^{1/2} \Delta D(N_B)_{s,k} - \frac{(n-1)N^{1/2}}{4(N-1)}\kappa_s.$$

Accordingly then, the following Theorem can be established:

Theorem 4.2 *Assuming that the DGP is given by (14) and (15) with $\tau_s = \kappa_s N^{\frac{1}{2}}$, then as $N \to \infty$, it can be determined that,*

i) $\frac{1}{\sqrt{N}}\widetilde{y}_{s[rN]} \Rightarrow \sigma\left[V_s(r) + \kappa_s d(\lambda, r) - \frac{1}{4}\sum_{s=1}^{4} r\kappa_s\right]$;

ii) $\frac{1}{N^{\frac{3}{2}}}\sum_{n=2}^{N}\widetilde{y}_{s,n} \Rightarrow \sigma\left[\int_0^1 V_s(r)dr + \kappa_s\lambda - \frac{1}{8}\sum_{s=1}^{4}\kappa_s\right]$;

iii) $$\frac{1}{N^2}\sum_{n=2}^{N}(\widetilde{y}_{s,n})^2 \Rightarrow \sigma^2\left[\int_0^1 V_s^2(r)dr + 2\kappa_s\int_\lambda^1 V_s(r)dr - \frac{1}{2}\sum_{s=1}^{4}\kappa_s\int_0^1 rV_s(r)dr \right. \\ \left. +\kappa_s^2\lambda - \frac{1}{2}(1-\lambda^2)\kappa_s\sum_{s=1}^{4}\kappa_s + \frac{1}{12}\sum_{s=1}^{4}\kappa_s^2\right];$$

iv) $$\frac{1}{N}\sum_{n=2}^{N}\Delta\widetilde{y}_{s,n}\widetilde{y}_{s,n-1} \Rightarrow \sigma^2\left[\int_0^1 V_s(r)dV_s(r) + \kappa_s\left[V_s(1) - V_s(\lambda)\right] - \frac{1}{4}\sum_{s=1}^{4}\kappa_s\int_0^1 rdV_s(r) \right. \\ \left. +\kappa_s V_s(\lambda) + \kappa_s^2 - \lambda\frac{2}{4}\kappa_s\sum_{s=1}^{4}\kappa_s - \frac{1}{4}\sum_{s=1}^{4}\kappa_s\int_0^1 V_s(r)dr + \frac{1}{2}\left(\frac{1}{4}\sum_{s=1}^{4}\kappa_s\right)^2\right]$$

where $V_s(r) = W_s(r) - rW(1)$ and $W(1) = \frac{1}{4}\sum_{s=1}^{4}W_s(1)$.

Consult working paper version (Hassler and Rodrigues (2001)) for a detailed derivation.

Remark 4.2: Similarly to what was observed in Theorem 4.1 for the HEGY test, Theorem 4.2 shows that also the limiting distributions of LM-HEGY test are affected by breaks that increase with $N^{1/2}$, but not by breaks of finite magnitude.

4.3 Misspecification of the Breakpoint

The importance of the new break corrected LM-HEGY test emerges from the obscurity of the true breakpoint which is conditioned by conjecture or estimation. It may be the case that the true breakpoint occurs at $T_B = \lambda T$, though assumed to occur at $T_A = \ell T$ with $\lambda \neq \ell$. As the results from Section 3 suggest, misspecification does not affect the limiting distribution as long as τ_s are bounded.

Theorem 4.3 *Assume that the DGP is (10) and (11) with $\alpha = 1$, and the true breakpoint $T_B = \lambda T$, while the estimation of (16) bears the breakpoint $T_A = \ell T$ with $\lambda \neq \ell$. Then, as $T \to \infty$, the results from Theorem 4.1 and 4.2 still hold true.*

The proof of this Theorem follows the same line of thought as the proof of Theorem 4.1 in Hassler and Rodrigues (2001).

Remark 4.3: This theorem implies that our new procedure is asymptotically valid as long as the breaks are bounded even if the exact breakpoint is not known a priori. In short, the time of the break may be known a priori, estimated from the data, or guessed. But even if we get the breakpoint wrong, the limiting results are robust in terms of misspecification. This robustness is well reproduced for finite samples in the Monte Carlo experiments, presented in the following Section.

5 Monte Carlo Evidence

The model simulated is a quarterly seasonal AR(1) process with additive outliers at time $T_B = \lambda T$ of the type,

$$y_t = x_t + \sum_{s=1}^{4} \tau_s D(T_B)_{s,t},\ x_t = \alpha\, x_{t-4} + \varepsilon_t,\quad \varepsilon_t \sim i.\mathcal{N}(0,1),\ t = 1, 2, \ldots, T. \tag{23}$$

If $\alpha = 1$, then the null hypothesis (1) holds true. Similarly, as in Lopes (2001), the mean shifts are modelled by one parameter τ. We consider six cases:

(i)	$\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau$,	(iv)	$\tau_1 = \tau_2 = \tau$, $\tau_3 = \tau_4 = 0$,
(ii)	$\tau_1 = \tau$, $\tau_2 = \tau_3 = \tau_4 = 0$,	(v)	$\tau_1 = \tau_2 = \tau_3 = \tau$, $\tau_4 = 0$,
(iii)	$\tau_1 = -\tau_2 = \tau$, $\tau_3 = \tau_4 = 0$,	(vi)	$\tau_1 = -\tau_2 = \tau_3 = -\tau_4 = \tau$.

In case (i), all seasons are affected by the same break, while in case (ii) only one season is affected. In cases (iii) and (iv), two neighbouring seasons are affected, with equal or opposite sign; in the latter case, the effect on the overall means is zero. In case (v), all seasons but one are affected identically, and in case (vi), all neighbouring seasons are affected with opposite sign so that there is no effect on the overall mean.

Table 1 contains 5% experimental levels under the null of $\alpha = 1$ for $\lambda = 0.5$ and $\tau = 3$. The results for $T = 100$ and $T = 200$ rely on 5000 replications performed by means of GAUSS. Rejection frequencies for the original HEGY test are based on (3); the finite sample critical values for $H_0^{(1)}$ and $H_0^{(2)}$ are taken from the Dickey-Fuller percentiles in MacKinnon (1991), and for $H_0^{(3,4)}$ from HEGY (1990). Rejection frequencies for the LM-HEGY test are based on (8); the finite sample critical values for $H_0^{(1)}$ are taken from Schmidt and Phillips (1992), for $H_0^{(2)}$ from the Dickey-Fuller percentiles in MacKinnon (1991), and for $H_0^{(3,4)}$ from HEGY (1990). The HEGY-AO and HEGY-IO tests rely on (4) and (5) with critical values by Smith and Otero (1997) and Franses and Hobijn (1997) for $T = 1000$ and $T = 80$, respectively. The break corrected LM-HEGY test (denoted by LM-HEGY-AO for convenience) builds on (8) with (9) and the same finite sample critical values as the LM-HEGY test.

The results from Table 1 for $\tau = 3$ (which is a moderate break of three times the standard deviation) can be summarised as follows. The innovative outlier model is clearly not appropriate in the case of additive outliers. In most cases, the HEGY-IO test results in rejection frequencies above 10% while the nominal level is 5%. Although the HEGY and LM-HEGY tests are asymptotically robust to additive outliers, these can reflect severe size problems in finite samples. In particular, for cases (i), (v) and (vi), we observe that the tests are very conservative at frequencies zero or $\pi/2$, while they may concurrently overreject at other frequencies. For cases (iii) and (iv), the tests are slightly conservative at frequencies π and zero, respectively. As the sample size grows from $T = 100$ to $T = 200$, the experimental size is closer to the nominal one, although strong distortions can still be found. From further simulations performed but not reported here[1] we determine that the deficiencies of these tests are not removed for break points different from $\lambda = 0.5$. However, the additive outlier correction from Smith and Otero (1997) and the new LM-HEGY-AO test, equally perform well in all cases. In general, because Smith and Otero (1997) provide percentiles only for $T = 1000$, the new test presents closer results to the nominal level while the Smith-Otero test overrejects.

[Insert Table 1 about here]

The power of the tests for $\alpha = 0.8$ in (23) was also examined (excluding the HEGY-IO test), in the possible event of better power properties resulting from HEGY and LM-HEGY tests, where they are conservative under the null. However, Table 2 establishes that this is not the case. Every time that these tests show to be conservative under the null they are less powerful than the break corrected LM-HEGY type test under the alternative. Moreover, the new test rejects most frequently at all frequencies in cases (ii) and (v). Finally, although the HEGY-AO test slightly overrejects under the null, there are only very few situations where it is more powerful than the LM-HEGY-AO test correcting for seasonal shifts. In summary: Tables 1 and 2 suggest that our new test outperforms its competitors in terms of size and power when testing for unit roots under additive seasonal mean shifts.

[Insert Table 2 about here]

We further considered the case of no seasonal means shifts, i.e. $\tau = 0$ in (23), see Table 3. The results suggest that the LM-HEGY is more powerful than the original HEGY test at frequency zero and that the LM-HEGY-AO test is more powerful than the HEGY-AO test at frequency zero and also at other frequencies when $T = 200$. Although the break correction in the absence of breaks should reduce the power in general, there are situations where LM-HEGY-AO is more powerful than HEGY or even LM-HEGY.

[Insert Table 3 about here]

Finally, we analysed a break fraction ℓ context assumed for the HEGY-AO and LM-HEGY-AO differing from the true one in (23), λ. Throughout, ℓ was chosen as 0.5, with

[1] These are available upon request.

computations involving $\lambda = 0.25,\ 0.4,\ 0.6,\ 0.75$. In all four cases very similar findings were observed. Table 4 reports only those results for $\lambda = 0.4$. It appears that the HEGY-AO is very sensitive with respect to an incorrect assumption on the breakpoint. In all cases except for (ii) gross size distortions are observed. In contrast, the LM-HEGY-AO test is very robust in the case of an incorrect assumption in relation to the true break fraction, thus supporting the asymptotic result obtained in Theorem 4.3. Hence, the new LM-HEGY-AO test can be applied in practice regardless whether the true breakpoint is exactly known or not.

[Insert Table 4 about here]

6 Conclusion

In this paper HEGY type seasonal unit root tests are investigated and a new test procedure is introduced. Using quarterly data for ilustrative purposes, we allow for seasonal mean shifts under the null hypothesis. Asymptotic evidence shows that the HEGY as well as a corresponding Lagrange Multiplier (LM) variant (LM-HEGY), recently proposed by Breitung and Franses (1998) and Rodrigues (2000), are not affected by mean shifts of finite magnitude. However, if the breaks in the seasonal means are allowed to grow with the sample size, then the limits do change. In this second case, the limit distributions of the HEGY and LM-HEGY statistics will yield poor approximations in finite samples with considerable mean shifts. Indeed, experimental observation determine that such tests are heavily biased in the presence of the structural breaks here considered. Depending on the assumed data generating process, the tests may reveal extremely conservative or alternatively oversized. For this reason, we require in practice tests that correct for seasonal means shifts.

In this paper, a given breakpoint is assumed. Franses and Hobijn (1997) proposed the inclusion of dummy variables to account for breaks in the original HEGY regression. However, in case of additive outliers, a Monte Carlo study shows that this procedure is not adequate. Smith and Otero (1997), on the other hand, suggest the removal of all deterministics from the levels of the series and apply the usual HEGY regression on the residuals. This procedure (denoted here as HEGY-AO) represents the assumption of additive outliers and the respective critical values depend on the breakpoint. As an alternative approach introduced in this paper, we suggest the removal of all deterministics from the *differences* of the series. The corresponding statistics are obtained from the Lagrange Multiplier principle, and the tests are defined as LM-HEGY-AO. Aside from clearing the structural breaks from the series, the new test is just as simple to perform as the original HEGY test. The limiting distributions are derived and shown not to depend on the true breakpoint, thus allowing for alternatives based on conjecture or estimation of the breakpoint. Furthermore, the necessary critical values are already conveniently available in the literature. By performing simulations, the new LM-HEGY-AO as well as the HEGY-AO have experimental sizes close to the nominal ones when the true breakpoint is known a priori. In terms of power, however, the HEGY-AO test is clearly outperformed by our new test. Moreover, the HEGY-AO test is far from the nominal size if the assumed breakpoint differs from the true one, while the LM-HEGY-AO test is robust to this kind of misspecification not only asymptotically but also in the experiments considered here.

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Table 1: 5% experimental level, $\tau = 3$, $\lambda = 0.5$

	$T = 100$			$T = 200$		
	t_{π_1}	t_{π_2}	F_{π_3,π_4}	t_{π_1}	t_{π_2}	F_{π_3,π_4}
	(i): $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau$					
HEGY	0.12	7.82	9.72	0.30	6.32	7.54
LM-HEGY	0.06	8.86	10.88	0.60	6.72	7.58
HEGY-AO	6.12	6.66	7.12	4.98	5.78	5.76
HEGY-IO	70.02	10.80	13.36	54.32	8.66	8.68
LM-HEGY-AO	4.40	4.64	5.90	4.62	5.06	5.40
	(ii): $\tau_1 = \tau$, $\tau_2 = \tau_3 = \tau_4 = 0$					
HEGY	4.12	3.82	4.66	4.70	4.04	4.98
LM-HEGY	4.18	4.36	4.88	4.26	4.56	4.68
HEGY-AO	6.00	6.50	7.36	5.60	5.86	6.72
HEGY-IO	12.08	11.68	14.94	9.94	9.34	8.98
LM-HEGY-AO	5.10	5.40	5.86	4.44	5.38	5.40
	(iii): $\tau_1 = -\tau_2 = \tau$, $\tau_3 = \tau_4 = 0$					
HEGY	6.78	2.42	4.12	5.76	2.84	4.62
LM-HEGY	7.44	2.50	4.80	6.14	3.00	4.78
HEGY-AO	5.70	6.74	7.08	5.58	5.62	6.16
HEGY-IO	9.28	28.22	22.56	7.52	18.74	12.94
LM-HEGY-AO	4.02	5.10	5.96	4.72	5.14	5.56
	(iv): $\tau_1 = \tau_2 = \tau$, $\tau_3 = \tau_4 = 0$					
HEGY	2.28	5.80	3.92	3.06	5.32	4.42
LM-HEGY	2.68	6.52	4.74	3.46	5.42	5.00
HEGY-AO	6.08	6.68	7.12	5.40	5.90	5.78
HEGY-IO	26.06	8.00	24.78	20.24	7.68	13.12
LM-HEGY-AO	5.30	5.30	6.02	4.68	4.74	5.50
	(v): $\tau_1 = \tau_2 = \tau_3 = \tau$, $\tau_4 = 0$					
HEGY	0.78	4.86	4.96	1.58	5.58	5.82
LM-HEGY	0.84	5.84	5.92	2.04	6.04	6.16
HEGY-AO	6.54	6.42	7.12	5.44	5.58	6.02
HEGY-IO	49.66	12.64	18.82	34.10	10.36	11.24
LM-HEGY-AO	4.66	5.22	5.44	4.82	5.20	5.34
	(vi): $\tau_1 = -\tau_2 = \tau_3 = -\tau_4 = \tau$					
HEGY	9.88	0.12	9.74	7.78	0.70	7.00
LM-HEGY	11.66	0.12	10.76	8.64	0.78	7.08
HEGY-AO	5.86	6.44	7.26	5.96	5.76	5.62
HEGY-IO	12.14	69.52	13.12	9.44	47.44	8.76
LM-HEGY-AO	4.88	5.30	5.76	4.68	4.64	5.50
	critical values					
HEGY	-3.45	-2.89	6.60	-3.43	-2.88	6.57
LM-HEGY	-3.06	-2.89	6.60	-3.04	-2.88	6.61
HEGY-AO	-4.02	-3.50	10.19	-4.02	-3.50	10.19
HEGY-IO	-3.64	-3.25	9.01	-3.64	-3.25	9.01
LM-HEGY-AO	-3.06	-2.89	6.60	-3.04	-2.88	6.61

Table 2: 5% experimental power, $\tau = 3$, $\lambda = 0.5$

	$T = 100$			$T = 200$		
	t_{π_1}	t_{π_2}	F_{π_3,π_4}	t_{π_1}	t_{π_2}	F_{π_3,π_4}
	(i): $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau$					
HEGY	0.28	21.52	38.66	1.58	45.10	76.86
LM-HEGY	0.34	23.12	40.80	2.33	46.18	77.06
HEGY-AO	6.36	12.64	17.68	11.54	22.64	39.22
LM-HEGY-AO	9.88	12.28	18.98	24.72	30.70	54.32
	(ii): $\tau_1 = \tau$, $\tau_2 = \tau_3 = \tau_4 = 0$					
HEGY	7.22	9.02	12.84	18.36	23.96	42.56
LM-HEGY	9.40	9.84	14.60	23.00	24.84	42.98
HEGY-AO	6.44	11.56	17.18	11.80	22.04	38.66
LM-HEGY-AO	10.50	11.84	18.86	24.62	30.30	54.02
	(iii): $\tau_1 = -\tau_2 = \tau$, $\tau_3 = \tau_4 = 0$					
HEGY	10.98	2.28	8.54	23.24	6.32	27.78
LM-HEGY	13.44	2.84	9.52	28.10	6.84	28.42
HEGY-AO	6.92	11.74	17.70	11.58	22.28	39.26
LM-HEGY-AO	9.18	11.96	19.32	23.94	29.66	51.74
	(iv): $\tau_1 = \tau_2 = \tau$, $\tau_3 = \tau_4 = 0$					
HEGY	3.38	15.94	8.62	10.88	38.80	28.65
LM-HEGY	4.32	16.84	9.78	14.90	39.40	28.60
HEGY-AO	6.50	11.80	17.56	11.14	22.40	38.80
LM-HEGY-AO	9.72	12.32	18.76	24.98	29.74	52.22
	(v): $\tau_1 = \tau_2 = \tau_3 = \tau$, $\tau_4 = 0$					
HEGY	1.14	11.28	17.30	4.92	27.20	50.60
LM-HEGY	1.60	12.14	18.74	6.40	27.44	50.42
HEGY-AO	6.62	11.24	18.04	11.32	22.24	39.30
LM-HEGY-AO	9.76	11.40	19.60	24.48	29.56	53.42
	(vi): $\tau_1 = -\tau_2 = \tau_3 = -\tau_4 = \tau$					
HEGY	14.80	0.00	35.92	26.26	0.10	74.68
LM-HEGY	17.96	0.00	38.72	33.36	0.08	75.06
HEGY-AO	6.62	11.38	17.78	10.68	22.70	38.22
LM-HEGY-AO	9.12	10.66	19.36	23.70	30.96	52.06

The above represent percentages of rejection at the nominal 5% level. The simulated model is (23) with $\alpha = 0.8$. (See text.)

Table 3: 5% experimental size and power, $\tau = 0$, $\lambda = 0.5$

	$T = 100$			$T = 200$		
	t_{π_1}	t_{π_2}	F_{π_3,π_4}	t_{π_1}	t_{π_2}	F_{π_3,π_4}
	$\alpha = 1$					
HEGY	4.32	4.54	5.06	4.46	3.96	5.16
LM-HEGY	4.42	4.70	5.54	4.66	4.04	5.46
HEGY-AO	5.82	6.38	7.18	5.32	5.14	6.06
LM-HEGY-AO	4.68	4.84	5.88	4.66	4.36	5.38
	$\alpha = 0.9$					
HEGY	5.20	6.84	10.06	7.44	11.30	19.54
LM-HEGY	5.44	7.56	11.14	8.96	12.02	20.04
HEGY-AO	4.52	7.88	10.76	4.78	9.34	13.30
LM-HEGY-AO	5.52	7.26	10.64	9.22	11.34	19.22
	$\alpha = 0.8$					
HEGY	7.98	12.12	21.14	20.24	34.02	63.38
LM-HEGY	9.82	13.28	22.02	25.68	34.12	63.34
HEGY-AO	6.54	11.18	18.54	10.88	21.50	39.72
LM-HEGY-AO	10.10	11.62	19.72	23.72	29.48	53.58
	$\alpha = 0.5$					
HEGY	36.50	60.74	89.60	95.42	99.70	100
LM-HEGY	45.80	61.02	89.24	94.28	99.58	100
HEGY-AO	27.60	46.66	73.76	82.46	95.52	99.96
LM-HEGY-AO	36.96	41.86	66.88	90.08	90.10	98.86

The above represent the percentages of rejection at the nominal 5% level. The simulated model is (23) with $\tau = 0$. (See text.)

Table 4: 5% experimental level, $\tau = 3$, $\lambda = 0.4$, $\ell = 0.5$

	$T = 100$			$T = 200$		
	t_{π_1}	t_{π_2}	F_{π_3,π_4}	t_{π_1}	t_{π_2}	F_{π_3,π_4}
	(i): $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau$					
HEGY-AO	0.08	12.38	17.20	0.44	8.64	9.68
LM-HEGY-AO	4.46	4.88	5.76	4.70	5.40	5.88
	(ii): $\tau_1 = \tau$, $\tau_2 = \tau_3 = \tau_4 = 0$					
HEGY-AO	5.84	5.88	6.84	5.98	5.90	5.68
LM-HEGY-AO	4.66	4.66	6.02	4.86	4.64	5.32
	(iii): $\tau_1 = -\tau_2 = \tau$, $\tau_3 = \tau_4 = 0$					
HEGY-AO	10.26	3.20	6.94	8.38	3.88	6.70
LM-HEGY-AO	4.60	4.76	5.96	4.88	4.60	4.94
	(iv): $\tau_1 = \tau_2 = \tau$, $\tau_3 = \tau_4 = 0$					
HEGY-AO	2.90	10.44	6.76	3.56	7.40	5.06
LM-HEGY-AO	4.58	4.94	6.28	4.92	5.42	5.66
	(v): $\tau_1 = \tau_2 = \tau_3 = \tau$, $\tau_4 = 0$					
HEGY-AO	0.84	9.32	11.64	1.34	7.32	7.38
LM-HEGY-AO	4.82	4.50	5.82	4.92	5.16	5.76
	(vi): $\tau_1 = -\tau_2 = \tau_3 = -\tau_4 = \tau$					
HEGY-AO	16.16	0.16	27.26	11.48	0.58	13.78
LM-HEGY-AO	5.10	4.90	6.32	4.98	5.08	5.62

The above represent the percentages of rejection at the nominal 5% level of the true null hypotheses of unit roots. The simulated model is (23) with $\alpha = 1$ and a true break fraction of $\lambda = 0.4$, while the assumed break fraction is $\ell = 0.5$. (See text.)